

February 1, 2008

# Modelling the light quark vacuum

A. Miranda

Department of Physics and Astronomy

University of Aarhus, DK-8000

E-mail:miranda@phys.au.dk

## Abstract

The triumphs of the Standard Model of Particle Physics call attention upon an old idea, that the so-called vacuum is an accessible physical medium, and not just a tautology.

I take this idea as a serious working hypothesis, and I suggest a new way of trying to define and understand the structure of the light quark vacuum within the general framework of the Standard Model.

PACS: 14.65.Bt

BCS-like theory of quark-lepton vacuum

## I. Introduction

We propose in this Letter that the physical vacuum, which has acquired a place of honour in contemporary physics because of the great empirical success of the Standard Model, is the "ground state" of a mostly unknown physical medium that can be probed just like any other physical medium - and therefore the idea is not just a tautology [1,2].

This idea has a long prehistory behind it. In modern physics, it first emerged as a possibility, suggested by detailed QED studies. We need only recall the Lamb shift, the anomalous magnetic moment of the electron, the Casimir effect. This repertoire became greatly extended by the Standard Model (the Higgs condensate, axial anomalies, radiative corrections of various types, the so-called confinement of colour - just to name the most famous).

Sections 2 and 3 introduce the Model Hamiltonian and Model Spaces assumed in this paper. Here the guidelines for constructing these are provided by the phenomenologically well-established Chiral Perturbation Theory of light mesons and baryons and the various chiral quark models [3,4]. The fundamental degrees of freedom are the chiral quarks of Perturbative QCD. The gluons are not represented in this model space, as it is established that there are no soft gluons. Neither are soft photons, as they are assumed to be weakly coupled to quarks. Therefore the proposals in this Letter are not immediately applicable to the lepton vacuum.

Sections 4 and 5 discuss the spectrum of u,d,s quasiparticles and the stability of the vacuum defined to support such quasiparticles. The empirical input is taken from the work of H.Leutwyler et al [5]. Particular attention is paid to spinless and flavourless  $q\bar{q}$  vacuum stability modes.

Section 6 suggests possible experimental checks through electroweak probings of the quark vacuum.

Finally, section 7 contains some concluding remarks.

## II. The Primary Model Hamiltonian $H^{(0)}$

The model Hamiltonian, to be used in connection with a 3-momentum model space  $\mathcal{D}$  for QCD quarks, is assumed to have the particular form:

$$H^{(0)}[\mathcal{D}] = U_B^{(0)} + H_0^{(0)} + V^{(0)}[\mathcal{D}] \quad (1)$$

If  $\Omega$  represents the quantization box volume, then  $U_B^{(0)}/\Omega$  is a "background energy density " as  $\Omega \rightarrow \infty$ , supposed to absorb the infinite zero-point energy of the virtual Dirac-Weyl quark seas. It is to be "trivially renormalized " away.

$$H_0^{(0)} = \sum_n \sum_\lambda \int d^3\vec{p} |\vec{p}| (\alpha_{n\vec{p}\lambda}^+ \alpha_{n\vec{p}\lambda} + \beta_{\bar{n}p\lambda}^+ \beta_{\bar{n}p\lambda}) \quad (2)$$

$$V^{(0)}[\mathcal{D}] = \sum_{n',n} \sum_{\lambda'\lambda} \int d^3\vec{p}' \int d^3\vec{p} < n'\lambda', \bar{n}'\lambda'; \vec{p}' | V | n\lambda, \bar{n}\lambda; \vec{p} > \alpha_{n'\vec{p}'\lambda'}^+ \beta_{\bar{n}'-\vec{p}'\lambda'}^+ \beta_{\bar{n}-\vec{p}\lambda} \alpha_{n\vec{p}\lambda}$$

We assume that both  $H_0^{(0)}$  and  $V^{(0)}$  are hermitean.

The basic degrees of freedom defining our model space are Weyl fields, or rather their Fourier components.

$n(\equiv u, d, s)$  stands for particle species with momentum  $\vec{p}$  and helicity  $\lambda$ ;  $\bar{n}(\equiv \bar{u}, \bar{d}, \bar{s})$  represent antiparticles, here defined to mean CP-conjugate states of quark states.

It is important to emphasize that interactions among the input Weyl fermions are defined only within a *momentum* space  $\mathcal{D}$  extending from zero up to about  $\Lambda_\chi = 1\text{GeV}$  (Section 5). This cut-off has therefore a *physical significance*: it cannot be removed without removing the model space itself!

Colour degrees of freedom are not *dynamically* involved in this model. Their role is indirect, as they basically determine the *choice* of the model Hamiltonian and the model spaces. Therefore they are not explicitly represented.

$\alpha_{n\vec{p}\lambda}^+$  ( $\alpha_{n\vec{p}\lambda}$ ) are Fourier components of Weyl fields creating (annihilating) a quark  $n\vec{p}\lambda$ .

$\beta_{\bar{n}p\lambda}^+$  ( $\beta_{\bar{n}p\lambda}$ ) are the corresponding entities for their antiquarks.

As we shall see, the general idea is to introduce a series of canonical transformations, starting from the fundamental Weyl fields, designed to successively diagonalize as much of  $H^{(0)}$  as possible. Small and neglected residual contributions thus defined could then (hopefully!) be added perturbatively, if necessary.

We begin by first choosing a Bogoliubov- Dirac canonical transformation (without loss of generality for the purposes of this paper [5]):

$$\begin{pmatrix} b_{n\vec{p}\lambda} \\ d_{\vec{n}-\vec{p}\lambda}^+ \end{pmatrix} = \begin{pmatrix} u_{np} & v_{np} \\ -v_{np}^* & u_{np}^* \end{pmatrix} \begin{pmatrix} \alpha_{n\vec{p}\lambda} \\ \beta_{\vec{n}-\vec{p}\lambda}^+ \end{pmatrix} \quad (3)$$

$$u_{np} = u_{\vec{n}p} \quad v_{np} = v_{\vec{n}p}$$

The new operator set  $b_{n\vec{p}\lambda}^+, d_{\vec{n}-\vec{p}\lambda}^+$  shall be referred to as "Dirac-Bogoliubov quasiparticle creation operators" or simply "DB-quasiparticles". Similarly for their annihilation counterparts. We define the "no particle state" [5] for these operators *at time t*:

$$b_{n\vec{p}\lambda}(t)|0\rangle_t = 0 \quad (4)$$

$$d_{\vec{n}-\vec{p}\lambda}(t)|0\rangle_t = 0 \quad (5)$$

The parameters  $u_{np}, v_{np}$  play the role of variational parameters at our disposal .

The following general condition upon the transformation matrix is imposed, ensuring its invertibility:

$$|u_{np}|^2 + |v_{np}|^2 = 1 \quad (6)$$

It is furthermore assumed (without loss of generality) that these (variational) parameters are real.

## A. Fermionic Modes

Given the Hamiltonian (1,2), the idea [5] is to solve the Heisenberg equations of motion for constants of motion  $\hat{\mathbf{K}}(t)$ :

$$i\frac{\partial}{\partial t}\hat{\mathbf{K}}(t) = [H^{(0)}, \hat{\mathbf{K}}(t)] \quad (7)$$

We begin by adjusting our variational parameters so that the quasiparticles generated by the Dirac-Bogoliubov transformation (3) are *stationary solutions* to the equations of motion (7) :

$$[H^{(0)}, b_{n\vec{p}\lambda}^+] = E_{np}b_{n\vec{p}\lambda}^+ + \dots \quad (8)$$

$$[H^{(0)}, d_{\bar{n}\bar{p}\lambda}^+] = E_{np}d_{\bar{n}\bar{p}\lambda}^+ + \dots \quad (9)$$

When computing the commutators, one finds both linear and non-linear terms of course, but *one extracts from the latter only linear contributions*, if any, and (at this stage) neglect the non-linear residuals.

Introducing the definitions

$$\mu \leftrightarrow (n\vec{p}\lambda) \text{ or } (n\vec{p}\lambda, \bar{n} - \vec{p}\lambda) \quad (10)$$

$$\Delta_\mu \equiv - \sum_{\nu \in \mathcal{D}} V_{\mu\nu} u_\nu v_\nu \equiv \Delta_\mu^* \quad (11)$$

$$F_\mu \equiv V_{\mu\mu} v_\mu^2 \equiv F_{\bar{\mu}} \quad (12)$$

$$e_\mu \equiv \varepsilon_\mu + F_\mu \quad (13)$$

we find that

$$E_\mu^2 = e_\mu^2 + \Delta_\mu^2 \quad (14)$$

$$v_\mu^2 = \frac{1}{2} \left( 1 - \frac{e_\mu}{E_\mu} \right) \quad (15)$$

$$u_\mu^2 = \frac{1}{2} \left( 1 + \frac{e_\mu}{E_\mu} \right) \quad (16)$$

This fixes the variational parameters .

The function  $e_\mu$  shall be referred to as "the corrected single particle energy" , for obvious reasons.

The function  $\Delta_\mu$  shall be referred to as "the gap function" , for reasons that should become clear in the sequel.

The function  $E_\mu$  shall be referred to as "DB-quasiparticle energy".

Self-consistency requires that all gaps be either positive or vanish. Solutions are discussed in Section 5.

### III. Model Hamiltonian $H^{(1)}$

We shall continue approximating by next considering another branch of vacuum excitations, more precisely, flavourless boson-like *vacuum excitation modes*. This should give some feeling about the *dynamical* stability of the above vacuum solutions of the equations of motion, at least in these most important channels .

Work continues within the framework of linearization procedures.

Again, we seek *stationary solutions* of the Heisenberg equation of motion :

$$i \frac{\partial}{\partial t} \hat{\mathbf{K}}(t) = [H^{(1)}, \hat{\mathbf{K}}(t)] \quad (17)$$

We find

$$\begin{aligned} H^{(1)} = & \text{const} + \sum_{\mu} E_{\mu} (b_{\mu}^{\dagger} b_{\mu} + d_{\bar{\mu}}^{\dagger} d_{\bar{\mu}}) + \{ \sum_{\mu, \nu} V_{\mu\nu} u_{\mu}^2 v_{\nu}^2 * b_{\mu}^{\dagger} d_{\bar{\nu}}^{\dagger} d_{\bar{\nu}} b_{\mu} + \\ & + \sum_{\mu, \nu} V_{\mu\nu} u_{\nu} v_{\nu} v_{\mu} u_{\mu} * b_{\nu}^{\dagger} d_{\bar{\mu}}^{\dagger} d_{\bar{\mu}} b_{\nu} + \sum_{\mu, \nu} V_{\mu\nu} (u_{\mu}^2 u_{\nu}^2 + v_{\mu}^2 v_{\nu}^2) * \Gamma_{\mu}^{\dagger} \Gamma_{\nu} \\ & - \sum_{\nu, \bar{\nu}'} V_{\mu\nu} u_{\mu}^2 v_{\nu}^2 * (\Gamma_{\mu}^{\dagger} \Gamma_{\nu}^{\dagger} + \Gamma_{\nu} \Gamma_{\nu}) \} + \sum_{\mu, \nu} g_{\mu\nu} \Gamma_{\nu}^{\dagger} (b_{\mu}^{\dagger} b_{\mu} + d_{\bar{\mu}}^{\dagger} d_{\bar{\mu}}) + h.c. \end{aligned}$$

where

$$\Gamma_{\rho}^{\dagger} \equiv b_{\rho}^{\dagger} d_{\bar{\rho}}^{\dagger}$$

$$g_{\mu\nu} \equiv V_{\mu\nu} u_{\mu} v_{\mu} (u_{\nu}^2 - v_{\nu}^2)$$

Once again we make a canonical transformation. Returning to the more detailed notation, we introduce

$$\begin{pmatrix} B_f(\vec{k}) \\ B_f^+(\vec{k}) \end{pmatrix} = \sum_{n\lambda} \int d^3\vec{p} \begin{pmatrix} X_{fn}(k, p) & -Y_{fn}(k, p) \\ -Y_{fn}(k, p) & X_{fn}(k, p) \end{pmatrix} \begin{pmatrix} \Gamma_{n\lambda}(\vec{p}) \\ \Gamma_{n\lambda}^+(\vec{p}) \end{pmatrix} \quad (18)$$

$$X_{fn}^2(k, p) - Y_{fn}^2(k, p) = 1 \quad (19)$$

$$[H^{(2)}, B_f^+(\vec{k})] = \omega_f(k) B_f^+(\vec{k}) \quad (20)$$

with the definitions

$$\Gamma_{n\lambda}^+(\vec{p}) \equiv b_{n\vec{p}\lambda}^+ d_{\vec{n}-\vec{p}\lambda}^+ \quad (21)$$

$$\omega_f(k) = \sqrt{k^2 + \omega_f(0)} \quad (22)$$

The next effective Hamiltonian  $\hat{H}^{(2)}[\mathcal{D}]$  is by definition the Hamiltonian that makes equations (20) exact. Solutions must satisfy condition (19). We illustrate the structure of the solutions in case of complete degeneracy  $m_u = m_d = m_s$ .

#### IV. On electroweak probings of the vacuum

The most straightforward way of confronting these (or similar) *ansatzes* on the quark vacuum with more or less direct experimental tests is to make use of the hitherto ignored couplings to the electroweak sector of the Standard Model [2,3].

We assume that, within the confines of Model Spaces chosen in this paper, these *model* couplings are given by the Model Hamiltonian

$$H^{(4)}[\mathcal{D}'] = H^{(3)}[\mathcal{D}] + V^{(3)} + H_0 + V_{ew} \quad (23)$$

where  $H_0$  describes free leptons and photons. "Residual" couplings are (2,3)

$$V^{(3)} = \sum_{\mu, \nu} g_{\mu\nu} \Gamma_{\nu}^+ (b_{\mu}^+ b_{\mu} + d_{\vec{\mu}}^+ d_{\vec{\mu}}) + h.c. \quad (24)$$

$$V_{ew} = e_0 \int d^3\vec{r} J_{\alpha}^{em}(\vec{r}, t) A^{\alpha}(\vec{r}, t) + \frac{G_V}{\sqrt{2}} \int d^3\vec{r} J_{\alpha}^{weak}(\vec{r}, t) l^{\alpha}(\vec{r}, t) + h.c. \quad (25)$$

The model space is now extended to  $\mathcal{D}'$  which includes in addition leptons and photons.

We make the usual ansatz for mesons (2,3):

$$|M; \vec{p}\rangle \equiv B_{M\vec{p}}^+ |0\rangle = \sum_{\nu_1 \oplus \bar{\nu}_2 = M} \int d^3\vec{p}_1 \int d^3\vec{p}_2 \quad (26)$$

$$\Psi_{M\nu_1\bar{\nu}_2}(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}) b_{\nu_1\vec{p}_1}^+ d_{\bar{\nu}_2\vec{p}_2}^+ |0\rangle$$

with appropriate normalization condition on the wavefunctions. Amplitudes for various electroweak processes can now be computed perturbatively [5].

## V. Parameters

The extensive analysis carried out by Leutwyler et al [6] revealed that the light quark obey the condition that their running masses in the  $\overline{MS}$  scheme at scale  $\mu = 1\text{GeV}$  must be

$$m_u = (5.1 \pm 0.9) \text{ MeV} \quad m_d = (9.3 \pm 1.4) \text{ MeV} \quad (27)$$

$$m_s = (175 \pm 25) \text{ MeV}$$

We use this information to fix our basic parameters.

Using the effective interaction

$$\langle n'\lambda', \bar{n}'\lambda'; \vec{p}' | V | n\lambda, \bar{n}\lambda; \vec{p} \rangle = -\frac{G_{n'n}}{8\pi\Lambda_\chi^2} \quad (28)$$

we find that the gap equations can be rewritten as

$$1 = G_1 I_u + G_2 \varkappa_1 I_d + G_3 \varkappa_2 I_s \quad (29)$$

$$1 = G_2 \frac{I_u}{\varkappa_1} + G_1 I_d + G_3 \frac{\varkappa_2}{\varkappa_1} I_s \quad (30)$$



$$1 = G_3 \left[ \frac{I_u}{\varkappa_2} + \frac{\varkappa_1}{\varkappa_2} I_d + I_s \right] \quad (31)$$

with the definitions

$$I_k = \Delta_k^2 f(1/\Delta_k) \quad k = u, d, s \quad (32)$$

$$f(u) = -\frac{1}{2} \ln[u + \sqrt{1+u^2}] + \frac{1}{2} u \sqrt{1+u^2} \quad (33)$$

We are thus simply trading the input (27) with the  $G$ 's, so that these numerically fix *all* our subsequent results and predictions.

We shall implement this kind of renormalization by following the example of a single gap equation:

$$\Delta = G * \int_0^1 dx x^2 u(x) v(x) = \Delta * \frac{G}{2} \int_0^1 dx \frac{x^2}{\sqrt{x^2 + \Delta^2}}$$

or

$$1 = \frac{G\Delta^2}{2} * f(1/\Delta) \quad (34)$$

Lowering the upper integration limit from 1 downwards to some value  $x$  we find that this gap equation is satisfied if the parameter  $G$  is properly adjusted:

$$1 = \frac{g(x)}{2} * \Delta^2 f(x/\Delta) \quad G = g(1) \quad (35)$$

The results are (u in units of 1Mev):

u	$G_1$	$G_2$	$G_3$
10	-0.3618	0.0167	0.2232
20	-0.0959	0.0030	0.0349
30	-0.0468	0.0012	0.0119

The question of the dynamical stability of these solutions shall be illustrated qualitatively. We thus ignore that the quasiparticles have in fact different gap parameters and set these to a common value:

$$\Delta_u = \Delta_d = \Delta_s = \Delta \quad (36)$$

$$G_1 = G_2 = G_3 = \bar{G}/3 \quad (37)$$

The answer boils down to showing that the determinantal equation

$$S(\omega) = \begin{vmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{vmatrix} = 0 \quad (38)$$

where

$$S_{11}(\omega) = 1 - \bar{G}(x_{\max})\mathbf{P} \int_0^{x_{\max}} du u^2 \frac{2E(u)}{(2E(u))^2 - \omega^2(k)} \quad (39)$$

$$S_{12}(\omega) = -\bar{G}(x_{\max})\omega(k)\mathbf{P} \int_0^{x_{\max}} du u^2 \frac{1}{(2E(u))^2 - \omega^2(k)} \quad (40)$$

$$S_{21}(\omega) = S_{12}(\omega) \quad (41)$$

$$S_{22}(\omega) = 1 - \bar{G}(x_{\max})\mathbf{P} \int_0^{x_{\max}} du u^2 \frac{2E(u)\alpha^2(u)}{(2E(u))^2 - \omega^2(k)} \quad (42)$$

with

$$\alpha(u) = \sqrt{1 - \left(\frac{\Delta}{E(u)}\right)^2} \quad (43)$$

has non-trivial solutions with  $\omega \geq 0$ .

Self energy corrections to DB-quasiparticles are not included. So  $\omega = 0$  is a solution, as long as the gap equation is satisfied. This is the Goldstone associated with the quantum mechanical breaking of the symmetry symbolized by conditions (39). The inclusion of self-energies would move this Goldstone away from zero. Ignoring self-energy corrections, we estimate that the non-Goldstone heavy boson has a rest mass (in units of 1GeV) roughly of the order

$$\omega^2(0) = 4\Delta^2 \left\{ 1 + \left[ \frac{1}{\Delta} \frac{\sqrt{1+\Delta^2} - \Delta}{\ln \frac{1+\sqrt{1+\Delta^2}}{\Delta}} \right]^2 \right\}$$

## VI. CONCLUDING REMARKS

The physics of chiral and flavour symmetries and symmetry breaking in the world of quarks and leptons, as assumed and described by the Standard Model [2], gives us a deeper "feeling" for the unknown medium to which one implicitly refers.

This Letter is inspired by the Chiral Perturbation Theory and the Chiral Quark Model (2,3) and asks how one could interpret their empirical successes in terms of quantum motions of underlying *QCD-quark fields in their (suitably defined) vacuum state*, at least for mesons that are apparently *strongly entangled with the quark vacuum*.

A model Hamiltonian (and associated model spaces) are chosen to contain *one kind of degrees of freedom only*, viz. the so-called *current quarks*, which we take to be spin- $\frac{1}{2}$  Weyl fermions. This input largely determines the choice of the vacuum *ansatz* and the appropriate Model Hamiltonian.

The weight in this article is put on quantum dynamics and not on symmetries. Thus isospin symmetry in particular is what it seems to be, i.e. an *hadronic* symmetry.

The simplest way to further test and develop these ideas is through electroweak probings (section 6).

## REFERENCES

- [1] T.D.Lee, Nucl.Phys.**A538**(1992)3c-14c.
- [2] G. E.Volovik, The Universe in a Helium Droplet (OXFORD University Press, 2003).
- [3] H. Georgi, Weak Interactions and Modern Particle Theory (Addison-Wesley Publishing Company, 1984).
- [4] L. Glozman, A reply to Isgur's critique, nucl-th/9909021
- [5] P.A.M. Dirac, a) Lectures on Quantum Field Theory, Academic Press, Inc., New York, 1966); b) in Nature **4941**, 115 (1964); c) Phys. Rev. **B139**, 684 (1965); d) Physics To-day **23**, 29 (1970).
- [6] H. Leutwyler, Light Quark Masses, arXiv: hep-ph/9609467v1, 25th September 1996.